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Satellite non-gravitational orbital perturbations and the detection of the gravitomagnetic clock effect

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Abstract

The general relativistic gravitomagnetic clock effect consists in the fact that two massive test bodies orbiting a central spinning mass in its equatorial plane along two identical circular trajectories, but in opposite directions, take different times in describing a full revolution with respect to an asymptotically inertial observer. In the field of the Earth such time shift amounts to 10^{-7} s. Detecting it by means of a space based mission with artificial satellites is a very demanding task because there are severe constraints on the precision with which the radial and azimuthal positions of a satellite must be known: $\delta r \sim 10^{-2}$ cm and $\delta\varphi \sim 10^{-2}$ milliarcseconds per revolution. In this paper we assess if the systematic errors induced by various non-gravitational perturbations allow to meet such stringent requirements. A couple of identical, passive laser-ranged satellites of LAGEOS type with their spins aligned with the Earth's one is considered. It turns out that all the non vanishing non-gravitational perturbations induce systematic errors in r and φ within the required constraints for a reasonable assumption of the mismodeling in some satellite's and Earth's parameters and/or by using dense satellites with small area-to-mass ratio. However, the error in the Earth's GM is by far the largest source of uncertainty in the azimuthal location which is affected at a level of 1.2 milliarcseconds per revolution.

1 Introduction

General Relativity, in its weak-field and slow-motion approximation, predicts that two massive test particles following in opposite directions two geodesic circular orbits of identical radius in the equatorial plane of a central spinning body take different times in describing a full revolution with respect to an asymptotically inertial observer. If t^+ denotes the period of the particle moving in the same sense of the rotation of the central body and t^- denotes the period of the particle moving in the opposite sense, it turns out that

$$\Delta t = t^+ - t^- = 4\pi \frac{J}{Mc^2}, \quad (1)$$

where J and M are the proper angular momentum and the mass, respectively, of the central source and c is the speed of light in vacuum. Note that the Newtonian gravitational constant G does not appear in eq.(1). The co-rotating particle turns out to be slower than the counter-rotating one. This is the so called gravitomagnetic clock effect [*Cohen and Mashhoon*, 1993; *Mashhoon et al.*, 1999; 2000]. Eq.(1) is also equal to the difference in the test particles' proper periods τ^\pm for test particles orbiting at distances larger than the gravitational radius $r_g = 2GM/c^2$ of the central body and neglecting terms of order $\mathcal{O}(c^{-4})$.

For a couple of artificial Earth satellites eq.(1) yields a time shift of 10^{-7} s which is large enough to be detected according to the present-day level of accuracy in timing measurements with atomic clocks [*Lichtenegger et al.*, 2000]. Note that eq.(1) is independent of the satellite's orbit radius. This feature could be fruitfully exploited in choosing suitably the orbital parameters of the satellites to be employed. However, it has been shown in [*Gronwald et al.*, 1997; *Lichtenegger et al.*, 2000] that, in order to make feasible such measurement, the radial and azimuthal positions of the satellites should be known at a level of $\delta r \sim 10^{-2}$ cm and $\delta\varphi \sim 10^{-2}$ milliarcseconds (mas) per revolution. Such constraints are very stringent: suffice it to say that the position of LAGEOS laser-ranged satellite is presently known at a centimeter level. Recently, in [*Iorio*, 2001; *Lichtenegger et al.*, 2001] the impact of the long-periodic and short-periodic gravitational perturbations, with the systematic errors induced by them in r and φ , have been worked out: the conclusion is that, at the present level of knowledge of the static and time-varying parts of the Earth's gravitational field [*Lemoine et al.*, 1998], the gravitational errors are larger than the required δr and $\delta\varphi$.

However, in the near future the new, more accurate data for the terrestrial gravitational field from the CHAMP and GRACE missions will be available and the situation could become more favorable. So, it appears important to assess the error budget due to the non-gravitational forces [Milani *et al.*, 1987] on the satellite's radial and azimuthal locations, which is the scope of this paper.

We will consider throughout it a couple of identical passive, geodetic, laser-ranged spherical satellites of LAGEOS type both because Satellite Laser Ranging (SLR) has reached in the last decade an astonishingly level of accuracy and because with this kind of satellites it is by far simpler to model accurately enough the non-gravitational perturbations. Indeed, they depend on the physical and geometrical characteristics of the satellites and on the geometry of their orbits as well. For example, twice a year, almost six months apart, the Sun, moving along the ecliptic, intersects the Earth's equatorial plane so that the satellite's revolutions occurring these times are affected by the phenomenon of eclipses. The radiative perturbations which are generated, directly or indirectly, by the solar electromagnetic radiation like the direct solar radiation pressure, the albedo and the Yarkovsky-Schach effect act differently on the satellite's orbit with respect to the orbital arcs described in full sunlight and in many cases such discrepancies are relevant and difficult to calculate. Moreover, the thermal forces which cause the Yarkovsky-Schach and the Rubincam effects depend on both the physical properties of the satellite and on the orientation of its spin axis with respect to an inertial frame. The terrestrial environment may cause the spin direction to change in a more or less predictable way over time spans of some years, as it is occurring to LAGEOS [Métris *et al.*, 1999].

In order to detect the gravitomagnetic time shift, which is cumulative, there is no need of very long orbital arcs: indeed, for example, the orbital period of LAGEOS amounts to 1×10^{-1} days only, so that $10^2 - 10^3$ revolutions [Lichtenegger *et al.*, 2000] would correspond to arcs $10^1 - 10^2$ days long¹. We could take advantage of this fact by choosing the arcs so to avoid the eclipses effects; moreover, over such short time spans it would be possible to consider the satellite's spin axis direction as fixed in the inertial space. Indeed, many thermal effects vanish for a suitable fixed spin axis direction.

¹However, as pointed out in [Iorio, 2001], short arcs would not allow to average out many gravitational perturbations

The basic assumptions of our study are the following:

- In order to simplify the calculations we will consider a couple of LAGEOS type satellites with semimajor axis $a = 12270$ km, as for LAGEOS, zero eccentricity e and inclination i and orbital period $P = 1.35 \times 10^4$ s
- The physical parameters of the satellites like area-to-mass ratio S/m , reflectivity and drag coefficients C_R and C_D , etc. are assumed to be equal to those of LAGEOS: $S/m = 6.87 \times 10^{-3}$ cm² g⁻¹, $C_R = 1.13$, $C_D = 4.9$
- We will assume that their spins are aligned with the Earth's spin axis assumed as z axis of a terrestrial equatorial inertial frame
- Only orbital arcs in full sunlight will be considered: the eclipses effects will be neglected
- The perturbations on r and φ will be averaged over an orbital revolution
- The perturbations which will be considered are the direct solar radiation pressure, the Earth's albedo, the Poynting-Robertson effect, the direct Earth IR radiation pressure, the anisotropic thermal radiation or Yarkovsky-Schach effect, the thermal trust or Rubincam effect, the atmospheric drag and the effect of the Earth's magnetic field.

The paper is organized as follows: in Section 2 and Section 3 the perturbations on r and φ , respectively, are worked out, while Section 4 is devoted to the conclusions.

2 The radial position

The radial position of a satellite in a perturbed circular orbit may change due to variations both in its semimajor axis a and its eccentricity e according to [Christodoulidis *et al.*, 1988]:

$$\Delta r = \sqrt{(\Delta a)^2 + \frac{1}{2}(a\Delta e)^2}. \quad (2)$$

The perturbations on a are calculated with [Milani *et al.*, 1987]:

$$\dot{a} = \frac{2}{n}T, \quad (3)$$

where $n = \frac{2\pi}{P}$ is the mean motion, P is the Keplerian satellite's orbital period and T is the along-track disturbing acceleration. Concerning the eccentricity, since we are dealing with circular orbits we will use the components of the eccentricity vector $h = e \cos \omega$, $k = e \sin \omega$ (ω

is the argument of perigee) and [Milani *et al.*, 1987]:

$$\dot{h} \equiv \dot{e} \cos \omega + \mathcal{O}(e) = \frac{1}{na} [-R \cos(\omega + f) + 2T \sin(\omega + f)] + \mathcal{O}(e), \quad (4)$$

$$\dot{k} \equiv \dot{e} \sin \omega + \mathcal{O}(e) = \frac{1}{na} [R \sin(\omega + f) + 2T \cos(\omega + f)] + \mathcal{O}(e), \quad (5)$$

where f is the true anomaly and R is the radial component of the perturbing acceleration. For a circular orbit, from eq.(4) and eq.(5) it follows:

$$\dot{e} = \sqrt{(\dot{h})^2 + (\dot{k})^2}. \quad (6)$$

2.1 The radiative perturbations

The perturbing acceleration due to direct solar radiation pressure can be approximately written as [Lucchesi, 1998]:

$$\mathbf{w}_{\odot} = -w_{\odot} \hat{s}, \quad (7)$$

where \hat{s} is the unit vector from the Earth to the Sun and:

$$w_{\odot} = \frac{S}{m} \frac{I_0}{c} C_R. \quad (8)$$

In eq.(7) the term $(d_{\odot}/r_{\odot})^2$, in which d_{\odot} is the semimajor axis of the Earth orbit around the Sun and r_{\odot} is its instantaneous distance, has been set equal to one. I_0 is the solar constant. For LAGEOS type satellites eq.(8) amounts to almost $3.6 \times 10^{-7} \text{ cm s}^{-2}$.

Concerning the semimajor axis, it turns out that if the total solar radiation force acting on the spacecraft can be expressed in the general form $\mathbf{F}(\hat{s})$, no long-term effect in a will appear to any order in e [Milani *et al.*, 1987]. So, $\langle \dot{a} \rangle_{2\pi} = 0$.

Regarding the eccentricity, it can be proved that it is affected, at zero order in e , by long-term periodic perturbations with an almost yearly period [Lucchesi, 1998]. The amplitudes for the components of the eccentricity vector are:

$$\langle \dot{h} \rangle_{2\pi} \propto \frac{3}{2} \frac{w_{\odot}}{na}, \quad (9)$$

$$\langle \dot{k} \rangle_{2\pi} \propto \frac{3}{2} \frac{w_{\odot}}{na}, \quad (10)$$

so that, over an orbital revolution, the radial position of the satellite changes of almost 11 cm [Milani *et al.*, 1987].

Assuming that the solar constant is known at 0.3% [Ciufolini *et al.*, 1997], the major source of uncertainty in the satellite's radial position due to the direct solar radiation pressure would reside in the C_R satellite's coefficient. Assuming a global 0.5% mismodeling, the systematic error would amount to $\delta r_{SRP} = 5 \times 10^{-2}$ cm. This result suggests that, in order to meet the requirement of $\delta r \sim 10^{-2}$ cm, a careful analysis of the optical properties of the satellites should be conducted in accurate pre-launch controls.

For a circular and equatorial orbit the perturbations on the satellite's radial position induced by the Earth's albedo are entirely due to the changes in the eccentricity vector as well. Indeed, the along-track acceleration, which is particularly sensitive to the specular part of Earth's albedo and to its spatial and temporal variations, can be expanded in terms of long-periodical harmonics whose coefficients are proportional to $\sin i$ [Anselmo *et al.*, 1983]. Concerning the eccentricity vector, the simple analytic model by Métris *et al.*, [1997], based on an uniform mean albedo of $\overline{\mathcal{A}} = 0.3$, turns out to be adequate. The related perturbation amounts to 8×10^{-2} cm per orbit. Then, the systematic error due to the mismodeling in $\overline{\mathcal{A}}$, assumed to be 10% [Lucchesi, 1998], amounts to 8×10^{-3} cm per orbit.

It may be important to note that a way to reduce the impact of the direct solar radiation pressure and the albedo could be the use of a couple of dense satellites with small $\frac{S}{m}$.

The Poynting-Robertson acceleration [Burns *et al.*, 1979] leaves unaffected the eccentricity while changes the semimajor axis with long-term perturbations whose nominal amplitudes are of the order of $10^{-3} - 10^{-4}$ cm per orbit. Consequently, the related systematic errors are negligible.

2.2 The thermal perturbations

Concerning the direct Earth IR radiation pressure, the Earth thermal emissivity \mathcal{E} can be modelled in a form of a latitude-dependent spherical harmonic expansion on a spherical Earth surface [Sehnal, 1981]:

$$\mathcal{E} = \mathcal{E}_0 + \mathcal{E}_1(t) P_1(\sin \phi) + \mathcal{E}_2 P_2(\sin \phi) + \dots, \quad (11)$$

where ϕ is the terrestrial latitude. Regarding the effects of the first two zonal constant terms \mathcal{E}_0 and \mathcal{E}_2 , $\langle \dot{a} \rangle_{2\pi} = 0$ since it is proportional to $(e \sin i)^2$ and $\langle \dot{e} \rangle_{2\pi} = 0$ since it is proportional

to $e(\sin i)^2$ [Sehna, 1981]. The perturbations due to $\mathcal{E}_1(t)$, which shows an approximately yearly variation, vanish as well since R and T are proportional to $\sin i$ [Métris et al., 1997].

The Yarkovsky-Schach effect [Afonso et al., 1989] induces perturbations on the semimajor axis which vanishes since the first non-zero term in \dot{a} is of order $\mathcal{O}(e)$ for orbital arcs in full sunlight. As far as the eccentricity vector is concerned, e would be perturbed at a level of 9×10^{-2} cm per orbit. However, such effect depends on the orientation of the satellite's spin axis $\hat{\xi}$: for a fixed direction such that $\xi_x = \xi_y = 0$, as it could be feasible over short arcs, $\langle \dot{h} \rangle_{2\pi} = \langle \dot{k} \rangle_{2\pi} = 0$ [Lucchesi, 1998].

The thermal thrust or Rubincam effect [Rubincam, 1987] depends on $\hat{\xi}$ as well; for $\xi_x = \xi_y = 0$ it vanishes because it can be proved that, in this case, R and T are proportional to $\sin i$.

2.3 Other perturbations

The neutral and charged drag has negligible effects on r . Indeed, by neglecting the corotation of the exosphere, it turns out that [Milani et al., 1987]:

$$\langle \Delta a \rangle_{2\pi} = -C_D \frac{S}{m} a^2 \varrho, \quad (12)$$

in which ϱ is the atmospheric density. For $\varrho = 8.4 \times 10^{-21}$ g cm⁻³ [Afonso et al., 1985] we would have a decay in a of 4.2×10^{-4} cm per revolution. The same holds for the charged drag [Rubincam, 1990]. For circular orbits the eccentricity is not affected by the drag [Milani et al., 1987].

If the satellites carry an electric charge, as it is the case for LAGEOS, the Earth's magnetic field acts upon them via the Lorentz force. Its effect on the semimajor axis is of course zero since the Lorentz force does not change the satellite's total mechanical energy W and, consequently, a : indeed, $W = -\frac{GM}{2a}$. Regarding the eccentricity vector, it turns out that $\langle \dot{h} \rangle_{2\pi} = \langle \dot{k} \rangle_{2\pi} = 0$ because R is constant and T is of order $\mathcal{O}(e)$.

3 The azimuthal position

As in [Iorio, 2001; Lichtenegger et al., 2001], for a satellite in an equatorial and circular orbit the rate of the azimuthal angle can be calculated by means of:

$$\dot{\varphi} = \dot{\omega} + \dot{\Omega} \cos i + \dot{\mathcal{M}}. \quad (13)$$

In eq.(13) Ω is the longitude of the ascending node and \mathcal{M} is the mean anomaly whose rate equation is given by:

$$\dot{\mathcal{M}} = n - \frac{2}{na} R \frac{r}{a} - \sqrt{1-e^2}(\dot{\omega} + \cos i \dot{\Omega}). \quad (14)$$

For $e = i = 0$ eqs.(13)-(14) yield:

$$\dot{\varphi} = n - \frac{2}{na} R. \quad (15)$$

In dealing with eq.(15) the indirect effects on n induced by the perturbations on a must be considered as well. At the first perturbative order they are: [Milani *et al.*, 1987]

$$\Delta n = -\frac{3n}{2a} \Delta a. \quad (16)$$

3.1 The radiative perturbations

The direct solar radiation pressure does not affect φ because, as seen in previous section, $\langle \dot{a} \rangle_{2\pi} = 0$; moreover, $\langle R \rangle_{2\pi} = 0$.

Regarding the Earth's albedo, the indirect perturbations on n vanish because $\langle \dot{a} \rangle_{2\pi} = 0$. According to the model by Métris *et al.*, [1997] $\langle R \rangle_{2\pi} = 0$.

The non-vanishing perturbations of order $\mathcal{O}(e^0)$ induced by the Poynting-Robertson effect, which amount to almost 10^{-5} mas per revolution or less are negligible.

3.2 The thermal perturbations

Concerning the direct Earth IR radiation pressure, $\langle \dot{a} \rangle_{2\pi} = 0$ so that the indirect perturbations on the mean motion vanish as well. The first two constant zonal terms of the Earth IR emissivity yield non-vanishing terms of zero order in e . Indeed, it turns out that, for \mathcal{E}_0 :

$$\langle -\frac{2}{na} R \rangle_{2\pi}^{(0)} = -\frac{2}{na} \left(\frac{R_{\oplus}}{a}\right)^2 \mathcal{E}_0 \frac{S}{m} \frac{C_R}{c}, \quad (17)$$

which yields -1.6×10^{-1} mas per revolution. The systematic error induced by the mismodeling in \mathcal{E}_0 is within the limit $\delta\varphi \sim 10^{-2}$ mas per revolution and could be reduced by using satellites with small area-to-mass ratio. The bias due to \mathcal{E}_2 is negligible since its nominal perturbation amounts to 1×10^{-3} mas per revolution. As in for r , also in this case $\mathcal{E}_1(t)$ does not contribute since R is proportional to $\sin i$.

The Yarkovsky-Schach effect does not affect the azimuthal position since $\langle \dot{a} \rangle_{2\pi} = 0$ and it turns out that R averages out over an orbital revolution. The same holds for the Rubincam effect which is not present when $\xi_z = \pm 1$ and $i = 0$.

3.3 Other perturbations

The indirect perturbation on n due to the drag shift experienced by the semimajor axis is negligible because it amounts to 6×10^{-4} mas per revolution, while:

$$\langle -\frac{2}{na}R \rangle_{2\pi} = C_D \frac{S}{m} nae \langle \sin E \rangle_{2\pi} = 0, \quad (18)$$

where E is the eccentric anomaly.

The effect of the Earth's magnetic field is completely negligible since it is of the order of 10^{-5} mas per revolution.

The major source of error in the azimuthal position turns out to be the Earth's GM . Indeed, $\Delta\varphi^{(GM)} = \frac{\delta(GM)}{2na^3} \times P = 1.2$ mas per revolution by assuming $\delta(GM_{\oplus}) = 8 \times 10^{11} \text{ cm}^3 \text{ s}^{-2}$ [McCarthy, 1996].

4 Conclusions

In the context of the gravitomagnetic clock effect, by considering a couple of identical SLR satellites of LAGEOS type with $e = i = 0$ and $\xi_x = \xi_y = 0$, $\xi_z = \pm 1$ over short orbital arcs in full sunlight, it turns out that some non-gravitational perturbations on r and φ vanish.

Regarding the radial position, the largest perturbations are due to the direct solar radiation pressure which would change the satellite's distance of almost 10^1 cm per revolution and the Earth's albedo which would induce a change of 8×10^{-2} cm per revolution. However, their systematic errors induced by the mismodeling in the optical properties of the satellites and the Earth's albedo should fall below the cutoff $\delta r \sim 10^{-2}$ cm. The influence of the other non vanishing perturbations, like Poynting-Robertson effect and neutral and charged drag, is negligible. The thermal perturbations vanish.

The azimuthal angle is perturbed by the Earth's IR direct radiation pressure at a level of 10^{-1} mas per revolution. The other non vanishing perturbations are negligible.

It should be pointed out that all the non-vanishing perturbations are proportional to $\frac{S}{m}$. This means that the impact of their systematic errors could be reduced by using particularly dense satellites with small area-to-mass ratio. However, the largest source of error in φ is the uncertainty in the Earth's GM which induce a bias of 1.2 mas per revolution. This is a hard limitation to overcome because it is independent of the particular satellite employed and is related to our knowledge of the terrestrial gravitational field. Moreover, as pointed out in [Iorio, 2001], at the present level of knowledge of the terrestrial gravitational field, the mismodeled gravitational time varying perturbations induce systematic errors which are larger than the required constraints; they could be overcome by averaging them over adequately long time spans.

The results of the present work suggest that for a suitable choice of the satellites to be employed and of their orbital geometry it would be possible to keep the non-gravitational perturbations within the required constraints in order to make feasible the measurement of the gravitomagnetic clock effect. As far as the systematic errors induced by the forces acting upon the satellites are concerned, the major problems come from the Earth's gravitational environment. Improvements in satellite tracking accuracy of almost two orders of magnitude are needed as well.

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